Estimating Telephone Demand: Validation of Previous Models

Robin Lee, Internal Revenue Service

Since 1965, the IRS has provided American taxpayers tax law information and assistance through a toll-free telephone system. During the 1994 filing season alone (January-April), IRS received almost 87.5 million toll-free calls and answered about 19 million of them (22 percent). The General Accounting Office (GAO) reports this percentage of calls answered as an accessibility measure to the Congress every filing season. The media then tells the public that IRS is answering only one of every five callers. However, equating the number of *calls* to the number of *callers* results in multiple counting of some people and underestimation of the percentage of taxpayers served, because some callers make repeated attempts until they get through.

The IRS has been interested in estimating true demand, the actual number of individuals trying to access the toll-free telephone system in any given time period. The IRS uses this information to determine the staffing and circuitry requirements. Much of the past effort to estimate true demand has been focused on converting the number of unanswered calls to the number of unique individuals by modeling the redial probability as a function of the blocking probability, using the data available from the automated system, such as the number of calls attempted, completed, blocked, and abandoned (Lee, 1993).

Beginning in October of 1993, U.S. Sprint, the IRS 800-number carrier, started producing the Unique Number Reports (UNR) that count the number of unique phone numbers each week. The IRS does not receive the actual Automated Number Identification (ANI) records, but only the aggregated counts, to assure the taxpayers of the confidentiality of their phone conversation with assistors.

The purpose of this study is to summarize the UNR findings on taxpayers' calling behaviors and to test the validity of the IRS' previous models and assumptions. The first section describes the IRS phone system and the measurement data collected from the system. The second section is an overview of the earlier approaches that IRS took to estimate true demand and the latest unique number method. The results are presented in the last section, followed by the implications on future study.

■ IRS Telephone System

When a taxpayer calls the IRS toll-free number, 800-tax-1040, the local phone company sends the call to the Sprint switching center. Sprint, then routes the call to one of the 32 answering sites, based upon a predetermined routing plan that matches the area codes of originating calls to the call sites.

Each call site is equipped with an Automatic Call Distributor (ACD), that receives and distributes the calls to open lines. If a call comes in while all the phone lines are tied up, the ACD rejects the call and the caller receives busy signals. These calls are referred to as *overflows* or *blocked calls.*

If a call gets connected, one of two things can happen. The call goes to an assistor right away or is put on hold until the next assistor becomes available. In sites with Voice Response Units (VRU), the callers with a touch-tone phones can route themselves to the assistors who specialize in the tax law area of their questions. If a caller hangs up after the call is connected before speaking to a live assistor, the call becomes an *abandoned call.*

The ACD reports can capture the number of total calls attempted (T), completed (C), blocked (B), and abandoned (AB) in any given time period. The number of net calls answered is the calls completed minus the abandoned (C-AB). These ACD statistics for both local and toll-free circuits combined are reported on the Taxpayer Service's weekly Telephone Data Reports (TDR).

■ IRS Methodologies: Past & Current

Earlier Methods

Over the years, the IRS has used two basic approaches to estimate true demand. One approach expresses demand as the sum of completed calls and a portion of blocked calls, representing the callers who stop trying, since taxpayers have two options -- keep trying until connected or give up. If we denote the redial probability as r, the proportion of blocked calls that are not retried is (1-r)B and demand can be written as C+(1-r)B. A major difficulty of this approach is estimating r.

The first formula of this type IRS used was the one-third formula, which was simply adding one-third of the unanswered calls to the number of calls answered. The one-third figure was developed on the basis of data provided by the telephone company's survey of people calling operators for directory listings during the 1950s. This approach of assuming the constant redial probability, regardless of how congested the phone system was not realistic.

The next formula, Slide, was developed by Carl Harris of George Mason University. His original formula, specified the redial rate as a linear function of blocking probability. This was later modified into a set of four formulas, each to be used at a different level of blocking to reflect a non-linear relationship between the redial and blocking probability (Harris et al., 1987). The assumption was that as the system gets congested, more callers will have to redial; and that a smaller percentage of unanswered calls represents unique callers. In 1988, the GAO compared these methods: one-third, Slide, and revised Slide and found that all three methods produced the similar estimates (GAO, 1989).

The second approach is known as the Treasury method. This method defines demand as the number of callers making their first attempt to reach the IRS on a particular issue. The method estimates the proportion of first time callers by asking a sample of taxpayers if they have received busy signals before. This sample-based estimate of the proportion of first time callers is multiplied by all the call attempts, in order to estimate the first time callers in the total population (Opitz, 1988). The formula is applied to hourly survey and ACD data. The daily and weekly demand estimate is the sum of the hourly estimates.

One advantage of this second approach is that the redial probability need not be estimated, because it counts demand when callers make the first try, as opposed to the last try, as in the first approach (Stone, 1989). The model's assumption, however, that the proportion of first time callers among the completed is the same as that among the blocked and total calling population holds only under a very restrictive condition -- the probability of a call getting blocked is constant within each sampling window (Lee, 1993).

New Method

The source of Unique Number Reports is called the Automatic Number Identification records. ANI records display date, area code, telephone number, call time, call length, call disposition, and destination. Demand is defined as the total number of unique first attempts. With this definition, demand estimate can be greater than the unique phone number counts if multiple calls are completed from the same number. Multiple completions from the same number are counted as multiple demand, to account for numbers with multiple users or multiple questions.

Table 1 is a partial listing of an accessibility report for the week ending April 16, 1994. Unique number demand was about 2.9 million. Of those, about 925 thousand calls got through and almost 2 million calls were blocked on the first try. The lost caller column shows how many of these blocked callers dropped out after making only one, two, or three attempts during the same week. The Cumulative Percent Complete is the cumulative number of connected calls divided by demand. So, you can say that 32 percent of callers got through in one try and 40 percent got through in two tries and so on. The bottom figure of that column is an overall weekly accessibility measure, meaning that 62 percent of

Table 1.--Unique Number Accessibility Report for Week Ending April 16, 1994

Number Attempted	Thousands of Calls				Percent of Calls		
	Total	Completed	Incompleted	Lost	Cumulative Completed	Blocked	Redial
1	**2,906**	**925**	**1,980**	**535**	**32**	**68**	--
2	1,446	234	1,212	211	40	84	73
3	1,001	147	854	109	45	85	83
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
Total	10,027	1,809	8,218	1,095	62	82	87

Count of unique telephone numbers	2,215,941
Average number of attempts per caller	3.5
Average number of contacts per caller	1.3

callers got in eventually by the end of that week. The last two columns are conditional blocking and redial probabilities. They show that 68 percent of calls got blocked on the first attempt and 73 percent of them made the second attempt.

The width of the window is crucial in determining the attempt level. It is possible that some of the 1.1 million lost callers on this table retried the following week, where they are again counted as first time callers. The effect of window length on demand estimate is later examined by comparing a one-week window to a four-week window.

■ Notations, Formulas, and Data

Notations

T:	Total Call Attempts
C:	Connected Calls
B:	Blocked Calls (Busies)
AB:	Abandoned Calls on Hold
CA:	Calls Answered (C-AB)
UA:	Unanswered Calls (B+AB)
T_1:	Total Calls on First Attempts
C_1:	Completed Calls on First Attempts
r:	Overall Redial Probability
r_i:	Redial Probability After the ith Unsuccessful Attempt
ß:	Overall Blocking Probability
$ß_i$:	Blocking Probability for the ith Attempt

Formulas

One-third Formula:

$$\text{Demand} = CA + UA/3$$

Slide Formula:

$$\text{Demand} = CA + UA*.31 \quad \text{when } UA \leq CA,$$

$$\text{Demand} = CA + .6(UA*C)/T \quad \text{when } CA < UA \leq 3CA,$$

$$\text{Demand} = CA/(.81-(.04*UA/CA)) \quad \text{when } 3CA < UA \leq 5.5CA,$$

$$\text{Demand} = CA + .126*UA \quad \text{when } UA > 5.5CA.$$

Treasury Formula:

$$\text{Demand} = T * (C_1/C)$$

Data

The data analyzed in this paper came from the Unique Number Reports during the 1994 filing season, January 2 through April 30. In order to compare these formulas to unique number demand, the Slide and Treasury demand had to be recalculated for the toll-free calls only. The Slide and Treasury demand estimates were already available on Telephone Data Reports but they measured both toll-free and local line demand. The Slide and Treasury formula requires the number of net calls answered, which is the connected calls minus the abandoned calls. One problem was that the number of abandoned calls are not reported on UNR. This is because ANI data only keep track of whether a call is initially connected or not. The telephone data on TDR have the number of first and secondary gate abandons, but the toll-free abandons are not separated from the local line abandoned calls.

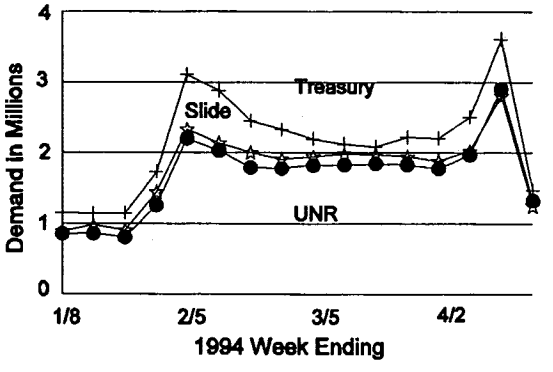
If a large percentage of connected calls is abandoned, then the number of completed calls on UNR will overestimate the number of answered calls. To account for the abandons, the weekly call abandon rates were estimated from the TDR and used to calculate the toll-free net calls answered, by assuming the toll-free and local lines have the same abandonment rate. The number of unanswered calls, thus was the number of Incompleted calls on UNR plus the number of abandoned calls estimated from TDR.

The percentage of abandoned calls reported on TDR ranged from 11.5 percent to 17 percent. But for most weeks, the abandonment rates were around 12 percent. The second to the last week of April had the highest percentage of abandons, at 17 percent.

■ Comparison of Formulas

Figure 1 shows the unique number demand and the Slide and Treasury demand estimates obtained from TDR, before they are adjusted for the difference in the coverage of calls. The Slide and Treasury lines should be higher than UNR, because they include local line demand, which is not measured on UNR. The figure doesn't tell us which formula is more accurate compared to the unique number method.

Figure 1. Comparisons of 1994 Filing Season



One observation worth mentioning, however, is that all three lines have a strikingly similar trend. The parallel lines suggest that there is a high correlation among the three methods graphed here. This is an interesting finding. The previous attempts to compare the two methods -- Slide and Treasury -- failed to show the evidence of any systematic relationship. The results of comparing the Slide and Treasury recomputed for the toll-free lines only are presented in Table 2.

Table 2.--Estimated Toll-Free Demand for 1994 Filing Season

Week Ending	UNR Demand	Slide	New Treasury	C+(1-r)B with r=.87
Jan. 8	852,501	738,495	866,229	779,586
Jan. 15	858,787	822,889	876,880	812,481
Jan. 22	877,621	789,824	916,503	841,588
Jan. 29	1,401,450	1,239,483	1,454,378	1,310,765
Feb. 5	2,202,104	2,141,235	2,287,576	2,283,601
Feb. 12	2,034,576	2,020,248	2,122,833	2,169,281
Feb. 19	1,793,214	1,720,033	1,873,974	1,861,065
Feb. 29	1,783,010	1,698,840	1,867,412	1,847,079
Mar. 5	1,827,097	1,671,426	1,901,183	1,795,682
Mar. 12	1,832,775	1,813,867	1,885,526	1,800,355
Mar. 19	1,850,688	1,862,145	1,878,122	1,818,772
Mar. 26	1,833,344	1,882,432	1,850,234	1,805,610
Apr. 2	1,777,837	1,724,790	1,797,841	1,751,178
Apr. 9	1,971,906	1,849,860	1,994,030	1,930,491
Apr. 16	2,905,543	2,668,874	2,942,297	2,877,722
Apr. 23	1,327,932	1,264,447	1,345,580	1,315,110
Total	28,341,471	26,991,028	29,087,926	28,175,561

For this comparison, the Treasury formula was replaced by the new Treasury formula, also developed by Dan Opitz, who proposed the Treasury method. This formula had a limited use until now, because it is based upon the conditional redial and blocking probabilities that were not measurable. The formula works like the Slide, in that the discount rate of total calls goes up as blocking goes up, to count a smaller fraction of total calls as unique. The new Treasury formula is:

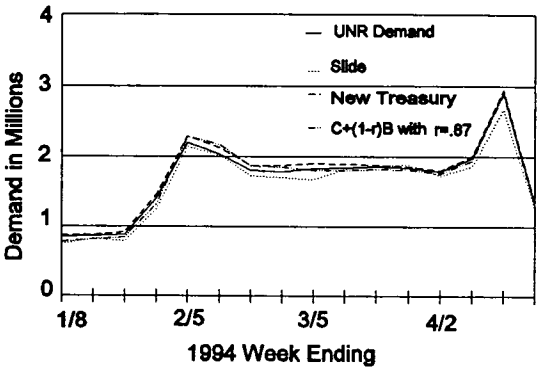
$$\text{Demand} = T / (1 + r_1\beta_1 + r_1r_2\beta_1\beta_2 + r_1r_2r_3\beta_1\beta_2\beta_3 + ...).$$

The Treasury survey method was not included in this comparison, because the formula requires collecting the data and calculating demand by hour. The hourly survey data were not available for this study. When the formula was applied to the weekly aggregate numbers, the method grossly overestimated demand. This, again, confirmed the earlier findings, that the method overestimates demand when the blocking rate is not constant in each window. The blocking rate is likely to fluctuate more in a weekly window than an hourly window.

As Table 2 shows, the new Treasury formula did better than Slide but worse than the basic formula -- C+(1-r)B. This is not surprising, because demand can be perfectly reproduced using the basic formula if the actual redial probability is available. The number is slightly off, because the average filing season redial probability of .87 was used, instead of real weekly redial rates. This implies that if the actual r is available, this formula is the one to use. For estimating local line demand, the toll-free redial rates, instead of the actual local line redial rates will be used. Thus, the accuracy of these formulas seems to depend on which redial probability -- between the overall and a set of conditional probabilities -- would be more stable, hence more transferable, from toll-free to local circuitry.

Figure 2 is a visual presentation of the same data presented in the Table 2. The graph illustrates that all of the methods have a similar trend.

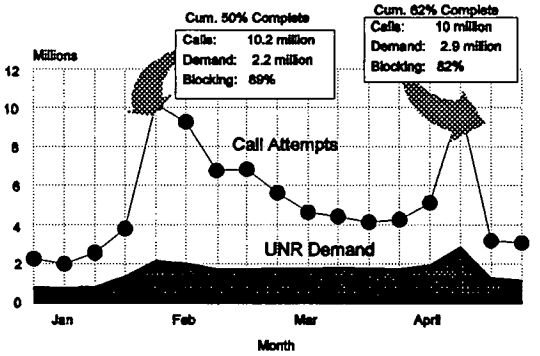
Figure 2. Estimated Demand on toll-Free Lines for 1994 Filing Season



■ Summary of UNR Findings

Data for the 1994 filing season are graphed in Figure 3.

Figure 3. Filing Season Demand & Call Attempts



Here are some highlights of what we have learned:

- The overall ratio of calls to true demand was 3.1 to 1. Among the 88 million toll-free calls, true demand was 28.3 million.
- Of the 28.3 million taxpayers, 19.2 million got through, which is 67 percent of all callers. This caller-based access measure is much higher than the caller-based measure of 22 percent that GAO uses.
- The average redial rate was .87 at the national level. This means that only 13 percent of the

blocked calls should be counted as unique individuals.

- The weekly redial probabilities were much more stable than blocking probabilities. The linear relationship between blocking and redial probability was only moderate.

- Highest demand came in the week ending April 16. The next highest was the week ending February 5. These two weeks illustrates very well why call volume doesn't measure demand accurately unless the blocking level is the same. The April week had 700,000 more demand than the February week, but had 160,000 fewer calls. The higher blocking rate of 89 percent for the February week compared to 82 percent for the April week resulted in more repeat calls and fewer unique callers.

- The slope of the call attempt line is steeper than that of demand, where it increases or decreases. This suggests that the call volume increases exponentially as demand increases.

- More than half the lost callers gave up after trying only once.

- The average number of contacts per caller increases as access improves. This phenomenon was labeled as "accessibility breeds access." It implies that the callers who got in easily are more likely to call back again with other questions.

- A one-week window compared to four-week overestimated demand by 2 percent (non-peak period) to 6 percent (peak period) and underestimated redial probability by 2 percent.

■ Conclusions and Future Study

Despite all these wonderful new findings, the unique number method has some shortcomings too. The method will overcount demand when callers use different phones to ask the same question or try again after abandoning on hold, retry in a different week, or get routed to a different call site at each attempt.

This study examined the accuracy of demand formulas and the underlying assumptions of the previously used formulas to find a suitable method for local demand estimation. The validity of using the toll-free redial probability reported on UNR to estimate local demand rests on the assumption that the local line redial probability is similar to that for the 800-number.

Currently the vendor is working on obtaining the ANI data from local Bell companies, so that the real data can be collected to test this assumption explicitly. The study will last for a year, to gather enough data to measure the variability of the redial probability across time and geographic area, in addition to the toll-free versus local lines. A final report is due at the end of 1995.

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