THE DESIGN OF SURVEYS USING MEASUREMENT DESIGN STANDARDS

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INTRODUCTION

This paper discusses the design of surveys which use a measurement design accuracy standard to assess the quality of data on events that occurred at some time in the past to sample persons. It is a sequel to an earlier paper by these same authors (Horvitz, et al., 1987) which argued the need to establish measurement design standards in order to assess the level of net systematic error, or bias, introduced by the conventional measurement designs used in human population surveys.

The earlier paper proposed that, in each and every sample survey, the net bias in the conventional measurement design being used be routinely estimated relative to the chosen accuracy standard. A "standard unbiased estimate" of the net bias generated by a particular conventional measurement design requires additional collection of the survey data with a comparable probability sample of the population of interest using the "standard measurement design". The difference between the estimate (e.g., proportion experiencing a specific event during the past year) obtained with the conventional measurement design and the estimate obtained when the standard measurement design is used is a design-based estimate of the net bias in the conventional measurement design relative to the chosen standard.

Finally, the earlier paper proposed that "survey statistics be routinely adjusted for measurement biases based on the chosen standards, just as they are now routinely adjusted to reduce coverage and nonresponse biases". No methods for such adjustments were given, however. The proposal to adjust for measurement biases based on chosen standards is expanded upon in the current paper. Specifically, an estimator, which combines the data collected using the "standard" measurement design with the data collected using the "conventional" measurement design, is proposed. The optimum survey design parameters, namely those that minimize total data collection costs while achieving a specified mean square error for the composite estimator, are determined. The optimum total sample size and the proportion of the total sample to be allocated to the standard measurement design have been computed for a range of survey conditions specified: first, by the bias ratio for the conventional measurement design; second, by the ratio of the variable unit costs of collecting the data for the standard design relative to the conventional measurement design; and, third, by the ratio of the unit variance for the standard design relative to the conventional design.

MEASUREMENT DESIGNS

A survey measurement design is represented by the specific set of factor levels which define the measurement process and which impinge upon the outcome of that process. For example, the mode of interview is a measurement factor which can occur in a given measurement design at one of three levels, namely, personal, telephone or mail. Factors and their levels which might appear in the measurement design for a survey gathering data on past events include:

FACTOR	LEVELS
Mode of Interview	Personal, telephone, mail
Respondent Rule	Self, proxy
Administration	Self, by interviewer
Interview Method	Paper and pencil, computer-assisted
Length of Recall	One month, two months, three months, etc.
Type of Recall	Bounded, unbounded

The choice of measurement design for a specific survey is usually dictated by cost and accuracy considerations, with cost often dominating, particularly in the absence of data on the ystematic error levels associated with alternative measurement designs. It is this lack of data on the net bias in conventional measurement designs that has prompted the proposal in the earlier paper that the survey research community adopt and use a single set of measurement design standards to estimate the measurement bias in survey estimates relative to the chosen standard design.

MEASUREMENT STANDARDS

An accuracy standard for measurements in surveys can be defined as that level for a given measurement factor which can be expected to yield the least biased data at the current state the art. For example, it is generally accepted that sample persons provide more accurate data about events occurring to themselves in the past than do proxy respondents. Thus, a standard unbiased estimate of the measurement bias generated by proxy respondents in a given survey is possible provided a design consistent probability sample of cases is selected for self-response measurement. The term "standard unbiased" refers to the accuracy of a measurement factor level relative to the chosen standard, which, in absolute terms, could still be biased.

Given a set of measurement standards, one for each of the measurement design factors, the bias for each of the other factor levels can be determined. By combining the set of measurement standards, a "standard measurement design" (SMD) is defined. For example, a consensus standard measurement design might be:

- Personal Interview;
- Computer Assisted;
- Self-Respondent; and
- One Month Bounded Recall.

Similarly, the actual or "conventional mea-

surement design" (CMD) used in a given survey can be defined as a combination of measurement factor levels. It might be, for example, a telephone survey, using paper and pencil, with proxy-as well as self-respondents, together with unbounded six-month recall. Although information on the components of the net measurement bias is important for establishing standards, it is not essential that the bias associated with each of the measurement design factor levels used in a specific survey be estimated in each and every survey. Rather, the net bias generated by a conventional measurement design can be estimated by also collecting the survey data for an independent design-consistent probability sample selected from the population of interest and using the SMD instead of the CMD.

ADJUSTED ESTIMATES

It has become increasingly routine to adjust survey estimates to reduce coverage and nonresponse biases. Although rarely explicitly stated, adjustments of sample weights for unit nonresponse, imputations for item nonresponse, and post-stratification adjustments are all dependent on acceptance of accuracy standards. Similarly, whenever survey data are collected using a standard measurement design in conjunction with a conventional measurement design, not only can the net measurement biases be estimated, but the conventional measurement design estimates can be adjusted to reduce, if not eliminate, their measurement biases. It is proposed to use a composite estimator for this purpose. The estimator and its properties, for simple random samples, are discussed briefly below and in greater detail in the appendix.

Assume that independent simple random samples of size n_c and n_s are selected, respectively, for the conventional and standard measurement designs. Assume further that the true mean of interest, $\mu + \tau$, is estimated by the sample mean, $\bar{y}_s$, for the SMD sample and that the CMD sample mean, $\bar{y}_c$, estimates $\mu - \tau$. Thus, $\bar{y}_c$ is a biased estimate of the true mean, with bias equal to -2τ. The variance of a single sample observation is assumed to be σ_s^2 and σ_c^2 for the respective measurement designs. Since σ_c^2 is likely to contain additional variance associated with the systematic error component of each CMD observation, σ_c^2 will usually be greater than σ_s^2.

The proposed estimator is

$$\hat{\mu} = (1-\lambda)(\bar{y}_s + \bar{y}_c)/2 + \lambda \bar{y}_s \qquad (1)$$

This is a biased estimator with bias equal to $-(1-\lambda)\tau$, which, in absolute value, is less than or equal to half the bias in $\bar{y}_c$. The value of λ which minimizes the mean square error of $\hat{\mu}$ is

$$\lambda_{OPT} = (\delta^2 + n_c^{-1} - Kn_s^{-1})/(\delta^2 + n_c^{-1} + Kn_s^{-1})$$

where

$$n_c \delta^2 = n_c (2\tau)^2 / \sigma_c^2$$

is the square of the bias ratio for $\bar{y}_c$ (i.e. ratio of the bias in $\bar{y}_c$ to the standard error of $\bar{y}_c$) and

$$K = \sigma_s^2 / \sigma_c^2$$

is the ratio of the unit variances. The minimum mean square error for $\hat{\mu}$ is then

$$MSE(\hat{\mu}) = \sigma_s^2 (n_c \delta^2 + 1)/n[(n_c \delta^2 + 1)\pi + K(1-\pi)] \quad (2)$$

where

$$\pi = n_s/(n_s + n_c) = n_s/n.$$

OPTIMUM COMBINED CONVENTIONAL AND STANDARD MEASUREMENT DESIGNS

In order to determine the best allocation of resources between the CMD and the SMD, the total survey cost

$$C = C_0 + n_c C_c + n_s C_s,$$

where C_c and C_s are the respective variable costs per sample unit for the conventional and standard measurement designs, was minimized subject to the requirement that the resulting

$$MSE(\hat{\mu}) = \sigma_s^2/m.$$

The optimum sample sizes are

$$(n_c)_{OPT} = (\sqrt{RK} - 1)/\delta^2 = m(\sqrt{RK} - 1)/\beta^2$$

and

$$(n_s)_{OPT} = m - \sqrt{K/R} (\sqrt{RK} - 1)/\delta^2$$

$$= m [1 - \sqrt{K/R} (\sqrt{RK} - 1)/\beta^2]$$

where $R = C_s/C_c$ is the variable cost ratio for an SMD observation relative to a CMD observation and $\beta^2 = m \delta^2$ is the square of the $\bar{y}_c$ bias ratio for samples of size m.

It follows that

$$(n/m)_{OPT} = 1 + (\sqrt{RK} - 1)(\sqrt{RK} - K)/\sqrt{RK} \beta^2$$

and

$$\pi_{OPT} = 1 - (\sqrt{RK} - 1)/[\beta^2 + (\sqrt{RK} - 1)(1 - \sqrt{K/R})].$$

The optimum designs for various combinations of the bias ratio β, the cost ratio R, and the variance ratio K are shown in Tables 1 and 2. Table 1 shows the optimum proportion of the total sample, that is $\pi = n_s/n$, to be measured using the SMD. Table 2 shows the total sample size inflation, n/m, for the optimum design. As expected, with the cost ratio R fixed, a greater proportion of the total sample is allocated to the SMD as the bias ratio β for $\bar{y}_c$ increases.

Table 1.   Sample Allocations for Survey Standards Subsampling ($\pi = n_s/n$)

Bias Ratio B	K	Cost Ratio						
		1.0	1.1	1.2	1.5	2.0	3.0	4.0
0.3	1.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.0	1.000	0.471	0.029	0.000	0.000	0.000	0.000
	0.8	1.000	1.000	1.000	0.175	0.000	0.000	0.000
	0.6	1.000	1.000	1.000	1.000	0.283	0.000	0.000
0.4	1.2	0.367	0.029	0.000	0.000	0.000	0.000	0.000
	1.0	1.000	0.699	0.433	0.000	0.000	0.000	0.000
	0.8	1.000	1.000	1.000	0.486	0.000	0.000	0.000
	0.6	1.000	1.000	1.000	1.000	0.530	0.021	0.000
0.5	1.2	0.604	0.388	0.200	0.000	0.000	0.000	0.000
	1.0	1.000	0.807	0.631	0.228	0.000	0.000	0.000
	0.8	1.000	1.000	1.000	0.654	0.237	0.000	0.000
	0.6	1.000	1.000	1.000	1.000	0.674	0.222	0.064
0.6	1.2	0.728	0.579	0.444	0.137	0.000	0.000	0.000
	1.0	1.000	0.865	0.741	0.440	0.139	0.000	0.000
	0.8	1.000	1.000	1.000	0.753	0.421	0.122	0.009
	0.6	1.000	1.000	1.000	1.000	0.763	0.378	0.211
0.7	1.2	0.802	0.692	0.592	0.351	0.105	0.000	0.000
	1.0	1.000	0.901	0.808	0.577	0.322	0.084	0.000
	0.8	1.000	1.000	1.000	0.815	0.549	0.273	0.148
	0.6	1.000	1.000	1.000	1.000	0.821	0.497	0.336
0.8	1.2	0.849	0.765	0.688	0.495	0.281	0.075	0.000
	1.0	1.000	0.924	0.853	0.670	0.456	0.229	0.123
	0.8	1.000	1.000	1.000	0.857	0.641	0.394	0.267
	0.6	1.000	1.000	1.000	1.000	0.860	0.588	0.438
0.9	1.2	0.881	0.815	0.753	0.596	0.412	0.213	0.117
	1.0	1.000	0.940	0.883	0.736	0.555	0.346	0.237
	0.8	1.000	1.000	1.000	0.886	0.708	0.489	0.367
	0.6	1.000	1.000	1.000	1.000	0.888	0.658	0.521
1.0	1.2	0.904	0.850	0.800	0.670	0.511	0.325	0.226
	1.0	1.000	0.951	0.905	0.784	0.631	0.441	0.333
	0.8	1.000	1.000	1.000	0.907	0.759	0.566	0.451
	0.6	1.000	1.000	1.000	1.000	0.909	0.713	0.589
1.5	1.2	0.957	0.934	0.911	0.851	0.769	0.652	0.573
	1.0	1.000	0.978	0.958	0.902	0.825	0.714	0.636
	0.8	1.000	1.000	1.000	0.958	0.887	0.782	0.706
	0.6	1.000	1.000	1.000	1.000	0.958	0.860	0.788

Table 2.   Sample Size Inflation Factor (n/m) for Survey Standards Subsampling

Bias Ratio B	K	Cost Ratio						
		1.0	1.1	1.2	1.5	2.0	3.0	4.0
0.3	1.2	0.901	0.901	0.901	0.901	0.901	0.901	0.901
	1.0	1.000	1.025	1.092	1.099	1.099	1.099	1.099
	0.8	1.000	1.000	1.000	1.000	1.408	1.408	1.408
	0.6	1.000	1.000	1.000	1.000	1.480	1.961	1.981
0.4	1.2	0.943	0.959	0.962	0.962	0.962	0.962	0.962
	1.0	1.000	1.014	1.052	1.190	1.190	1.190	1.190
	0.8	1.000	1.000	1.000	1.161	1.563	1.563	1.563
	0.6	1.000	1.000	1.000	1.000	1.270	2.180	2.273
0.5	1.2	0.964	0.974	1.000	1.053	1.053	1.053	1.053
	1.0	1.000	1.009	1.033	1.165	1.333	1.333	1.333
	0.8	1.000	1.000	1.000	1.103	1.389	1.818	1.818
	0.6	1.000	1.000	1.000	1.000	1.173	1.755	2.346
0.6	1.2	0.975	0.982	1.000	1.100	1.190	1.190	1.190
	1.0	1.000	1.006	1.023	1.115	1.337	1.563	1.563
	0.8	1.000	1.000	1.000	1.072	1.270	1.738	2.211
	0.6	1.000	1.000	1.000	1.000	1.120	1.525	1.935
0.7	1.2	0.981	0.986	1.000	1.074	1.253	1.408	1.408
	1.0	1.000	1.005	1.017	1.084	1.248	1.631	1.961
	0.8	1.000	1.000	1.000	1.053	1.199	1.542	1.890
	0.6	1.000	1.000	1.000	1.000	1.088	1.385	1.687
0.8	1.2	0.986	0.990	1.000	1.056	1.193	1.515	1.786
	1.0	1.000	1.004	1.013	1.064	1.190	1.483	1.781
	0.8	1.000	1.000	1.000	1.040	1.152	1.415	1.681
	0.6	1.000	1.000	1.000	1.000	1.067	1.295	1.526
0.9	1.2	0.989	0.992	1.000	1.045	1.153	1.407	1.665
	1.0	1.000	1.003	1.010	1.051	1.150	1.382	1.617
	0.8	1.000	1.000	1.000	1.032	1.120	1.328	1.538
	0.6	1.000	1.000	1.000	1.000	1.053	1.233	1.415
1.0	1.2	0.991	0.993	1.000	1.036	1.124	1.330	1.539
	1.0	1.000	1.002	1.008	1.041	1.121	1.309	1.500
	0.8	1.000	1.000	1.000	1.026	1.097	1.266	1.436
	0.6	1.000	1.000	1.000	1.000	1.043	1.189	1.336
1.5	1.2	0.996	0.997	1.000	1.016	1.055	1.147	1.239
	1.0	1.000	1.001	1.004	1.018	1.054	1.138	1.222
	0.8	1.000	1.000	1.000	1.011	1.043	1.118	1.194
	0.6	1.000	1.000	1.000	1.000	1.019	1.084	1.150

Also as expected, with fixed bias ratio, a smaller proportion of the total sample is allocated to the SMD as the cost of collecting data with the SMD increases relative to the CMD data collection cost. With R and β both fixed, a greater proportion of the total sample is allocated to the SMD as the unit variance σ_c^2 increases relative to σ_S^2; that is, as K decreases.

What is surprising in Table 1 is that a majority of the total sample is allocated to the SMD in about twice as many cells as for the CMD. This is clearly contrary to usual practice where there is often some reluctance to use a more costly, yet more accurate measurement design. In this context, Table 1 suggests that even when the measurement bias is rather small relative to the parameter being estimated, a sizeable proportion of the total sample should be invested in the SMD. For example, if there is only a two percent measurement bias with a CMD for a variable with a 100 percent coefficient of variation, then the bias ratio for various values of m is:

m	Bias Ratio
400	0.4
900	0.6
1600	0.8
2500	1.0
3600	1.2
6400	1.6.

This simple example, together with Table 1, suggests that rather small measurement biases can quickly dominate the MSE of estimates derived from a conventional measurement design and that, unless the cost ratio for the standard measurement design is prohibitively high, a significant proportion of the total sample should be allocated to the SMD.

When $\pi < 1$, the optimum design provides an estimate, $\hat{\mu}$, with the same mean squared error, but at less cost than a sample of size m devoted entirely to the SMD. In this situation, n>m and (1-π) is the proportion of the total sample assigned to the CMD. Table 2 reflects the additional sample, relative to m, which is assigned to the CMD. In general, the incremental sample increases as the cost ratio R increases, decreases as the bias ratio β increases, and decreases as the variance ratio K decreases.

LaVange and Folsom (1985) have computed victimization rates for personal crimes with contact adjusted to a standard measurement design model for the 1978 National Crime Survey (NCS). The SMD selected consisted of bounded, personal, self-response interviews at the second time in panel, and a recency distribution which weighted the effect of 1-3 month recall 1.75 times that of a 4-6 month recall to account for the joint effect of internal telescoping of events and memory loss biases. Accepting these adjusted victimization rates as standards, the estimated bias ratios for the 1978 NCS actual measurement design are shown in Table 3. The 1978 NCS measurement design included both telephone and personal interviews, proxy- as well as self-respondents, unbounded as well as bounded interviews, six months recall of victimization events, and interviews with the same respondents every six months up to a total of seven interviews.

Since the optimum design achieves the desired mean square error at least cost, it is of interest to determine the savings realized by the optimum design relative to using the SMD exclusively with sample size m. The percent cost savings for optimum designs defined by the same combinations of bias, variance and cost ratios as in Table 1 are shown in Table 4. As expected, the greatest relative savings occur when more of the total sample can be allocated to the CMD and when the cost ratio is high. There is little opportunity to save money with a CMD when its bias ratio is high.

EFFECT OF COMPLEX SURVEY DESIGNS

The optimum designs given in Tables 1 and 2 assume simple random sampling. However, these results should remain essentially the same for more complex survey designs involving stratified, multistage, cluster samples. The applicability of Tables 1 and 2 to more complex designs is most likely when both the CMD and SMD samples are independently generated using the same sampling frame and the same complex sample design since the design effects will then be the same, provided, of course, that the variance ratio K for the two measurement designs remains constant.

CONCLUSIONS

This paper continues to emphasize the need to recognize and assess the level of systematic error or net bias associated with the measurement process in human population surveys collecting data on events that occurred sometime in the past to sample persons. It looks at survey designs which would use measurement design standards to both determine and adjust for the net bias in conventional measurement design estimates.

The primary problem addressed is that of determining the least costly allocation of available resources between an inexpensive, but biased conventional measurement design and a more expensive, but less biased standard measurement design in order to realize a combined estimate which satisfies a mean square error constraint. The set of optimum designs in Tables 1 and 2 reinforces the need for survey measurement design standards to provide a basis for determining the net bias in conventional measurement designs, at least relative to the chosen standard. In fact, the optimum designs tend to allocate more than half of the total sample to the standard design except for situations in which the bias ratio for the conventional design estimate is less than 0.5 or the cost of collecting data with the standard measurement is at least 50 percent greater than with the conventional design and there is little added variance due to the systematic errors in the conventional design.

As stated in our earlier paper on the use of standards, "The survey research community can

Table 3. Estimated Bias Ratios for Victimization Rates 1978 National Crime Survey

Measurement Factor Levels Conventional	Standard	Personal Crimes with Contact	Personal Crimes without Contact
Proxy respondent	Self-response	-2.25	-1.61
Telephone Interview	Personal Interview	0.33	0.22
Recency			
6 months	1 month	-9.98	-13.28
6 months	2 months	-5.81	-8.94
6 months	3 months	-2.01	-5.01
Time-In-Panel			
7th interview	2nd interview	-1.90	-1.77
6th interview	2nd interview	-1.52	-1.20
5th interview	2nd interview	-1.22	-0.20
4th interview	2nd interview	-1.02	-0.26
3rd interview	2nd interview	-0.88	-0.05
Unbounded Interview	Bounded Interview	5.64	9.54
CMD vs SMD*			
1978			
1st Quarter		-4.58	1.95
2nd Quarter		-2.91	0.16
3rd Quarter		-5.31	-0.21
4th Quarter		-5.71	-0.92
Sex			
Male		-3.31	1.25
Female		-4.85	3.02
Race/Ethnicity			
Hispanic		1.48	1.83
Black, nonHispanic		2.72	2.49
White, nonHispanic		-7.62	1.54
Age			
16-19 years		0.34	1.06
20-24 years		4.17	9.69
25-34 years		2.40	9.53
35-49 years		-8.15	2.34
Family Income			
$10,000-14,999		-1.97	3.30
$15,000-24,999		-3.90	4.78
$25,000+		-3.24	8.72

* As defined in the text.

Table 4. Cost Savings Ratio for Survey Standards Subsampling

Bias Ratio/	Variance Ratio (R)	Cost Ratio						
		1.0	1.1	1.2	1.5	2.0	3.0	4.0
0.3	1.2	0.099	0.000	0.000	0.000	0.000	0.000	0.000
	1.0	0.000	2.406	8.435	0.000	0.000	0.000	0.000
	0.8	0.000	0.000	0.000	6.748	0.000	0.000	0.000
	0.6	0.000	0.000	0.000	0.000	5.061	0.000	0.000
0.4	1.2	5.694	12.599	0.000	0.000	0.000	0.000	0.000
	1.0	0.000	1.354	4.745	0.000	0.000	0.000	0.000
	0.8	0.000	0.000	0.000	3.796	0.000	0.000	0.000
	0.6	0.000	0.000	0.000	0.000	2.847	24.316	0.000
0.5	1.2	3.444	8.064	13.333	0.000	0.000	0.000	0.000
	1.0	0.000	0.866	3.037	13.469	0.000	0.000	0.000
	0.8	0.000	0.000	0.000	2.429	14.036	15.562	0.000
	0.6	0.000	0.000	0.000	0.000	1.822	0.000	30.161
0.6	1.2	2.530	5.600	9.259	21.615	0.000	0.000	0.000
	1.0	0.000	0.602	2.109	9.354	23.830	0.000	0.000
	0.8	0.000	0.000	0.000	1.687	9.747	27.927	43.215
	0.6	0.000	0.000	0.000	0.000	1.265	10.807	20.945
0.7	1.2	1.859	4.114	6.803	15.880	30.777	0.000	0.000
	1.0	0.000	0.442	1.549	6.872	17.507	0.000	0.000
	0.8	0.000	0.000	0.000	1.239	7.161	20.518	31.750
	0.6	0.000	0.000	0.000	0.000	0.930	7.940	15.388
0.8	1.2	1.423	3.150	5.208	12.158	23.564	41.941	0.000
	1.0	0.000	0.338	1.186	5.261	13.404	27.911	39.062
	0.8	0.000	0.000	0.000	0.949	5.483	15.709	24.308
	0.6	0.000	0.000	0.000	0.000	0.712	6.079	11.782
0.9	1.2	1.125	2.489	4.115	9.606	18.618	33.139	43.772
	1.0	0.000	0.267	0.937	4.157	10.591	22.053	30.864
	0.8	0.000	0.000	0.000	0.750	4.332	12.412	19.207
	0.6	0.000	0.000	0.000	0.000	0.562	4.803	9.309
1.0	1.2	0.911	2.016	3.333	7.781	15.081	26.842	35.455
	1.0	0.000	0.217	0.759	3.367	8.579	17.883	25.000
	0.8	0.000	0.000	0.000	0.607	3.509	10.054	15.557
	0.6	0.000	0.000	0.000	0.000	0.455	3.891	7.540
1.5	1.2	0.495	0.896	1.481	3.458	6.703	11.930	15.758
	1.0	0.000	0.096	0.337	1.497	3.813	7.939	11.111
	0.8	0.000	0.000	0.000	0.270	1.560	4.468	6.914
	0.6	0.000	0.000	0.000	0.000	0.202	1.729	3.351

benefit through the adoption of a single set of accuracy standards for controllable measurement design factors." This paper clearly demonstrates that it makes sense to know the bias, cost and variance ratios of estimates based on conventional survey measurement designs relative to estimates based on measurement designs defined by a set of adopted standards. Given a single set of standards and use of the measurement designs defined by these standards in conjunction with conventional measurement designs, valuable information on the critical bias, cost and variance ratios will be realized. This information will not only enable better allocation of resources between the conventional and standard designs, but the adjusted estimates should have greater accuracy, at least relative to the chosen standards.

REFERENCES

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APPENDIX

Our composite estimator has the form

$$\hat{\mu} = (1-\lambda)(0.5)(\bar{y}_S + \bar{y}_C) + \lambda\bar{y}_S$$

$$= (0.5)[(1+\lambda)\bar{y}_S + (1-\lambda)\bar{y}_C].$$

We assume that $\bar{y}_S$ is standard unbiased with expectation $(\mu+\tau)$ and that $\bar{y}_C$ has expectation $(\mu-\tau)$. The associated bias in $\hat{\mu}$ is

$$\text{Bias}(\hat{\mu}) = -(1-\lambda)\tau.$$

Assuming that $\bar{y}_S$ and $\bar{y}_C$ are independent, the variance of $\hat{\mu}$ is

$$\text{Var}(\hat{\mu}) = (0.25)[(1+\lambda)^2(\sigma_S^2 \div n_S) + (1-\lambda)^2(\sigma_C^2 \div n_C)].$$

Noting that σ_C^2 includes the variance of individual measurement biases and the covariance of these biases with the associated true values, we define $\sigma^2 \equiv \sigma_C^2$ and assume that $K = (\sigma_S^2 \div \sigma_C^2)$ is most commonly less than 1. Noting further that

$$\text{Bias}^2(\hat{\mu}) = (1-\lambda)^2 \tau^2$$

$$= (0.25) \sigma^2[(2\tau)^2 \div \sigma^2] (1-\lambda)^2,$$

we define $\delta \equiv [(2\tau) \div \sigma]$. The parameter δ can be viewed as the σ scaled absolute bias in $\bar{y}_C$ relative to the standard.

In terms of the notation defined above, we have

$$\text{MSE}(\hat{\mu}) = (0.25)\sigma^2[(1+\lambda)^2 Kn_S^{-1} + (1-\lambda)^2(\delta^2 + n_C^{-1})].$$

The value of λ, say λ_0, that minimizes $\text{MSE}(\hat{\mu})$ is

$$\lambda_0 = (\delta^2 - Kn_S^{-1} + n_C^{-1}) \div (\delta^2 + Kn_S^{-1} + n_C^{-1}). \quad (A.1)$$

The form of MSE based on λ_0 becomes

$$\text{MSE}_0 = (K\sigma^2 \div n_S)(\delta^2 + n_C^{-1}) \div (\delta^2 + Kn_S^{-1} + n_C^{-1}). \quad (A.2)$$

To specify optimum values for the sample sizes n_S and n_C, we minimize the simple linear survey cost function

$$C(n_S, n_C) = C_0 + n_S C_S + n_C C_C$$

subject to the mean-squared-error constraint

$$\text{MSE}_0(n_S, n_C) = (K\sigma^2 \div m).$$

Note that this MSE constraint requires that we achieve the MSE associated with the design ($n_S = m$, and $n_C = 0$).

We begin the solution by recasting the MSE constraint in the form

$$n_C(m\delta^2 - K) - n_S - n_C n_S \delta^2 + m = 0.$$

The associated lagrangian is therefore

$$F(n_C, n_S) = (C_0 + n_S C_S + n_C C_C) - \gamma[n_C(m\delta^2 - K) - n_S - n_C n_S \delta^2 + m].$$

Setting the derivatives to zero, the following three solution equations are obtained

$$C_S = -\gamma(n_C \delta^2 + 1) \quad (A.3)$$

$$C_C = -\gamma(n_S \delta^2 - m\delta^2 + K) \quad (A.4)$$

and

$$0 = n_C(m\delta^2 - K) - n_S - n_C n_S \delta^2 + m. \quad (A.5)$$

Dividing Eq. (A.3) by Eq. (A.4) yields the result

$$(C_S \div C_C) = (n_C\delta^2 + 1) \div (n_S\delta^2 - m\delta^2 + K)$$

Defining the cost ratio $R \equiv (C_S \div C_C) \geq 1$, we solve the equation above for n_S yielding

$$n_S = [(n_C\delta^2 + 1) + R(m\delta^2 - K)] \div R\delta^2. \quad (A.6)$$

Substituting Eq. (A.6) into the MSE constraint Eq. (A.5), one obtains after some simplification

$$(n_c \delta^2 + 1)^2 = RK.$$

This quadratic equation yields the following n_c solution

$$(n_c)_{opt} = (\sqrt{RK} - 1) \div \delta^2. \qquad (A.7)$$

Our solution for n_s is obtained by

substituting the n_c result from Eq. (A.7) into

Eq. (A.6). This substitution leads to

$$(n_s)_{opt} = (\sqrt{RK} + R\delta^2 m - RK) \div R\delta^2$$

$$= m - \sqrt{K \div R} \ (\sqrt{RK} - 1) \div \delta^2$$

$$= m - \sqrt{K \div R} \ n_c . \qquad (A.8)$$

Combining Eq.'s (A.7) and (A.8) we obtain the optimum allocation fraction $\tau = [n_s \div (n_s + n_c)]$

for the standard subsample as a function of the bias ratio parameter

$$\beta = \sqrt{m} \ \delta$$

$$= \sqrt{m} \ (2\tau) \div \sigma_c.$$

Note that β is [Bias $(\bar{y}_c)$ ÷ SE$(\bar{y}_c)$] when $n_c = m$.

In terms of this bias-ratio for the conventional mean $\bar{y}_c$ when $n_c = m$, we have

$$\tau_{opt} = 1 - \{(\sqrt{RK} - 1) \div [\beta^2 + (\sqrt{RK} - 1)(1 - \sqrt{K \div R})]\}. \qquad (A.9)$$

The optimum design's percent saving in variable survey costs relative to the design with $n_s = m$ and $n_c = 0$ is

$$\phi_{opt} = (100)[mC_s - (n_s C_s + n_c C_c)] \div (mC_s)$$

$$= (100)[(\sqrt{RK} - 1)^2 \div R\beta^2]. \qquad (A.10)$$

Recall that the design ($n_s = m$, $n_c = 0$) was used to

establish our MSE constraint.

Finally, it is interesting to note how much the optimum sample size $n = (n_s + n_c)$ is inflated

relative to the design ($n_s = m, n_c = 0$). This

inflation factor has the form:

$$(n \div m)_{opt} = 1 + [(\sqrt{RK} - 1)(\sqrt{RK} - K) \div \sqrt{RK} \ \beta^2]. \qquad (A.11)$$