

APPENDIX C: DERIVATION OF COST MINIMIZATION MODEL FRAMEWORK

To guide development of compliance cost models for what-if analysis, a simple economic model was developed based on the premise that a rational taxpayer will choose between preparing his return himself or seeking the services of a third party preparer depending on which choice minimizes tax burden, holding constant all other influencing factors. This economic model of compliance costs was developed based on work by Eichfelder and Schorn (2009).¹

The model consists of the following variables and parameters:

C_p	Time spent on tax preparation
R_p	Resources spent on tax preparation, including human capital and physical capital.
R_e	External resources expended on tax compliance (i.e., 3 rd party tax preparer)
$p_e R_e$	External assistance help cost
E	The entity's earnings
O_k	Entity's tax planning options, itemize vs. standard deductions or income shifting. Options include compliant and non-compliant tax planning options.
θ	Productivity of personal resources.
p_e	Competitive market price of external tax help

Compliance cost: Compliance cost consists of the individual/firm's personal burden which is a function of the individual's own resources plus hiring an outside tax specialist with a market price of external tax prep.

$$C = C_p(R_p) + p_e R_e$$

$C_p(R_p)$ is the individual/firm's time spent on tax preparation, in addition to the resources spent on tax preparation software/hardware.

¹Eichfelder, Sebastian and Michael Schorn, 2009. "Tax Compliance Costs: A Business Administration Perspective." Working Paper No. 2009/3. Free University Berlin, School of Business and Economics, Berlin, Germany.

Filing Activities: This is part of the constraint in the minimization model; the amount of activities that need to be done in order to file their taxes. The amount of activities needed depends on the entity's earnings and tax planning and tax reporting compliance.

$$A = A(E, O_k)$$

Resource Production: This is the other part of the constraint. The output produced by the resources used. θ is the production weight of personal resources relative to the production by external resources.

$$\theta R_p + R_e$$

Cost Minimization: A rational tax complier will choose the allocation of personal and third party resources to minimize total compliance cost constrained by the amount of activities needed for tax compliance.

$$\min_{R_p, R_e} \text{TotCost} = C_p(R_p) + p_e R_e \quad (1)$$

$$\text{Subject to the constraint } A(E, O_k) = \theta R_p + R_e \quad (2)$$

This can be solved using a Lagrangian multiplier. The gross marginal cost of in-house resources must equal the market price of external outsourced tax compliance activities.

$$L = C_p(R_p) + p_e R_e + \lambda [A(E, O_k) - \theta R_p - R_e] \quad (3)$$

First-order Conditions

$$\frac{\partial C_p}{\partial R_p} = \lambda \theta \quad (4)$$

$$p_e = \lambda \quad (5)$$

$$A(E, O_k) - \theta R_p - R_e = 0 \quad (6)$$

Second-order Conditions

$$H = \begin{vmatrix} C_p'' & 0 & -\theta \\ 0 & 0 & -1 \\ -\theta & -1 & 0 \end{vmatrix} < 0 \quad (7)$$

$$H = C_p'' \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} - 0 \begin{vmatrix} 0 & -1 \\ -\theta & 0 \end{vmatrix} + (-\theta) \begin{vmatrix} 0 & 0 \\ -\theta & -1 \end{vmatrix} = -C_p'' < 0$$

Hessian matrix of second derivatives must be negative for a minimization, so $C_p'' > 0$

From the First-order Conditions

Combining (1) and (2)

$$\frac{\partial C_p}{\partial R_p} = p_e \theta > 0 \quad (8)$$

With rational choice, the taxpayer chooses a cost-optimal mix of resources. The in-house marginal cost of tax reporting per resource equals the external market price of third party assistance. Also the shadow price of the constraint is equal to the market price of third party assistance $p_e = \lambda$. If the constraint loosens by one unit (of activities), burden increases by p_e .

We could solve this model to find functions of the cost minimizing resources:

$$R_i = R_i^*(\theta, p_e, E, O_k) \text{ where } i = p, e$$

$$\lambda = \lambda^*(\theta, p_e, E, O_k)$$

Identities

$$C_p'(R_p^*(\theta, p_e, E, O_k)) - \lambda^*(\theta, p_e, E, O_k) \theta \equiv 0 \quad (4')$$

$$p_e - \lambda^*(\theta, p_e, E, O_k) \equiv 0 \quad (5')$$

$$A(E, O_k) - \theta R_p^*(\theta, p_e, E, O_k) - R_e^*(\theta, p_e, E, O_k) \equiv 0 \quad (6')$$

$$TC_{\min} = C_p(R_p^*(\theta, E, p_e, O_k)) + p_e R_e^*(\theta, E, p_e, O_k) \quad (1')$$

Implications:

In order to see how total burden changes with changing parameters we need to look at how the cost minimizing resources $R_i^*(\theta, p_e, E, O_k)$ respond to the changing parameters θ, p_e, E, O_k . The minimized resources are linked by the identities (4'), (5') and (6')

Change in personal productivity: $\Delta\theta$

$$\begin{aligned}
 C_p'' \frac{\partial R_p^*}{\partial \theta} - \frac{\partial \lambda^*}{\partial \theta} \theta - \lambda^* &\equiv 0 \\
 -\frac{\partial \lambda^*}{\partial \theta} &\equiv 0 \\
 -\theta \frac{\partial R_p^*}{\partial \theta} - R_p^* - \frac{\partial R_e^*}{\partial \theta} &\equiv 0
 \end{aligned}
 \quad \text{matrix form} \quad \rightarrow \quad
 \begin{bmatrix} C_p'' & 0 & -\theta \\ 0 & 0 & -1 \\ -\theta & -1 & 0 \end{bmatrix}
 \begin{bmatrix} \frac{\partial R_p^*}{\partial \theta} \\ \frac{\partial R_e^*}{\partial \theta} \\ \frac{\partial \lambda^*}{\partial \theta} \end{bmatrix}
 =
 \begin{bmatrix} \lambda^* \\ 0 \\ R_p^* \end{bmatrix}$$

Using Cramer's Rule

$$\frac{\partial R_p^*}{\partial \theta} = \frac{\begin{vmatrix} \lambda & 0 & -\theta \\ 0 & 0 & -1 \\ R_p & -1 & 0 \end{vmatrix}}{H} = \lambda \frac{\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}}{H} - 0 \frac{\begin{vmatrix} 0 & -1 \\ R_p & 0 \end{vmatrix}}{H} + (-\theta) \frac{\begin{vmatrix} 0 & 0 \\ R_p & -1 \end{vmatrix}}{H} = \frac{-\lambda}{H}$$

$$\frac{\partial R_p^*}{\partial \theta} > 0 \text{ with } \lambda > 0; H < 0$$

$$\frac{\partial R_e^*}{\partial \theta} = \frac{\begin{vmatrix} C_p'' & \lambda & -\theta \\ 0 & 0 & -1 \\ -\theta & R_p & 0 \end{vmatrix}}{H} = C_p'' \frac{\begin{vmatrix} 0 & -1 \\ R_p & 0 \end{vmatrix}}{H} - \lambda \frac{\begin{vmatrix} 0 & -1 \\ -\theta & 0 \end{vmatrix}}{H} + (-\theta) \frac{\begin{vmatrix} 0 & 0 \\ -\theta & R_p \end{vmatrix}}{H} = \frac{C_p'' R_p}{H} + \frac{\lambda \theta}{H}$$

$$\frac{\partial R_e^*}{\partial \theta} < 0 \text{ with } C_p'' > 0; H < 0; R_p > 0; \lambda > 0; \theta > 0$$

$$\frac{\partial \lambda^*}{\partial \theta} = 0 \text{ from the identities.}$$

Implication 1. As personal productivity increases, holding all else constant, the taxpayer uses more personal resources and less third party resources.

Change in price of 3rd party tax help: ΔP_e

$$\begin{aligned}
 C_p'' \frac{\partial R_p^*}{\partial P_e} - \frac{\partial \lambda^*}{\partial P_e} \theta &\equiv 0 & \frac{\partial R_p^*}{\partial P_e} &\equiv \frac{\theta}{C_p''} > 0 \\
 1 - \frac{\partial \lambda^*}{\partial P_e} &\equiv 0 & \text{using algebra} & \frac{\partial \lambda^*}{\partial P_e} &\equiv 1 \\
 -\theta \frac{\partial R_p^*}{\partial P_e} - \frac{\partial R_e^*}{\partial P_e} &\equiv 0 & \frac{\partial R_e^*}{\partial P_e} &\equiv -\theta \left(\frac{\partial R_p^*}{\partial P_e} \right) = -\frac{\theta^2}{C_p''} < 0
 \end{aligned}$$

Implication 2. As the price of third party tax help increases, holding all else constant, the taxpayer uses more personal resources and less third party resources.

Change in earnings: ΔE

$$C_p'' \frac{\partial R_p^*}{\partial E} - \frac{\partial \lambda^*}{\partial E} \theta \equiv 0$$

$$- \frac{\partial \lambda^*}{\partial E} \equiv 0$$

$$\frac{\partial A}{\partial E} - \theta \frac{\partial R_p^*}{\partial E} - \frac{\partial R_e^*}{\partial E} \equiv 0$$

using algebra

$$\frac{\partial R_p^*}{\partial E} \equiv 0$$

$$\frac{\partial \lambda^*}{\partial E} \equiv 0$$

$$\frac{\partial R_e^*}{\partial E} \equiv \frac{\partial A}{\partial E} > 0$$

Implication 3. As a taxpayer's earnings increase, holding all else constant, the taxpayer solely relies on additional third party resources.

$$TC_{\min} = C_p \left(R_p^*(\theta, E, p_e, O_k) \right) + p_e R_e^*(\theta, E, p_e, O_k) \quad (1')$$

$$\Delta E : \quad \frac{\partial TC^*}{\partial E} = p_e \frac{\partial R_e^*}{\partial E} > 0$$

Implication 4. As earnings increase compliance costs increase.